Approssimazioni

$\langle M, S \rangle$

$\mathcal{L}_H$ ↑ H — S: sp. top. compatto

due tipi di formule atomiche

$\tau \in C$

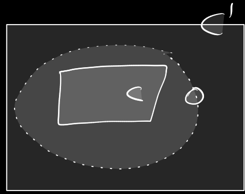
↑ k-tuple di termini — $C \subseteq S^k$

τ termine di sorta $H^n \times S^m \to S$

$\tau(x; y)$

formule atomiche in $\mathcal{L}_H$

$\varphi' > \varphi$ se le occorrenze di $\tau \in C$ in φ sono sostituite da $\tau \in C'$ dove $C' > C$



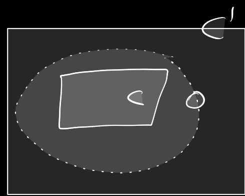
OSS 1 $\varphi \to \varphi'$

OSS 2 se $\varphi \in \mathcal{L}_H$ $\varphi > \varphi$

OSS 3 $\varphi' > \varphi$ esiste φ'' t.c. $\varphi' > \varphi'' > \varphi$

negazione forte

$\hat{\varphi} \perp \varphi$ se le occorrenze di $\tau \in C$ sono sostituite da $\tau \in \tilde{C}$ dove $\tilde{C} \cap C = \emptyset$



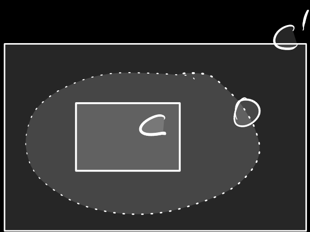
$\tau_\varphi \to \tau_{\neg\varphi}$ $\quad \forall \to \exists$ $\quad \wedge \to \vee$ $\quad \varphi \to \neg\hat{\varphi}$

$\exists \to \forall$ $\quad \vee \to \wedge$ $\quad \hat{\varphi} \to \neg\varphi$

Fatto ① $\varphi' > \varphi$ esiste $\hat{\varphi} \perp \varphi$ t.c. $\varphi \to \neg\hat{\varphi} \to \varphi'$

② $\hat{\varphi} \perp \varphi$ esiste $\varphi' > \varphi$ t.c. $\varphi \to \varphi' \to \neg\hat{\varphi}$

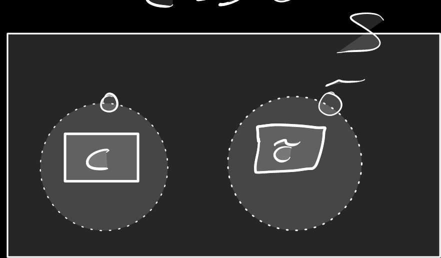
Dim ①



$\tilde{C} = C' \setminus O$

Dim ②

$C' = S \setminus \tilde{O}$



$\mathcal{F}^c \subseteq \mathcal{F}^p$

~~$\mathcal{L}_H$~~

Lemma N p-ω-satura $\varphi \in \mathcal{F}^p$

$N \models \{\varphi\}' \leftrightarrow \varphi$

dove $\{\varphi\}' = \{\varphi' : \varphi' > \varphi\}$

Dim $\varphi = (\tau \in C)$

$\{\varphi\}' = \{\tau \in C' : C' > C\} \leftrightarrow \tau \in \bigcap_{C' > C} C' = C$

Passo induttivo per esistenziale:

$\{\varphi(x)\}' \overset{IH}{\longleftrightarrow} \varphi(x)$ $\qquad$ $\{\exists x \varphi(x)\}' \longleftrightarrow \exists x \varphi(x)$ OBBIETTIVO

$\exists x \{\varphi(x)\}' \longleftrightarrow \exists x\, \varphi(x)$

Riposo TdM

in un modello sufficientemente saturo

$\exists y\; p(x,y) = \{\exists y\, \varphi(x,y) : \varphi(x,y) \in p\}$

Controesempio $\mathfrak{Z}^p$ $\quad S = [0,1] \quad M = [0,1] \quad i : [0,1]_M \longrightarrow [0,1]_S$ identità

$L_H = \{<\}$

$\varphi = \exists x [\underbrace{x \neq 0}_{L_H} \wedge i(x) \in \{0\}]$

$M \models \neg\varphi \qquad M \models \varphi'$

per ogni $\varphi' > \varphi$

M è p-massimale se $\exists N \succeq^p M \quad N \models \varphi \Rightarrow M \models \varphi$

se M p-massimale $\quad M \preceq^p N \Leftrightarrow (M \models \varphi \Leftrightarrow N \models \varphi)$

Lemma LSASE ① M p-massimale

② $M \models \{\varphi\}' \longrightarrow \varphi$

Dim ①⇒② $M \models \{\varphi\}'$ prendiamo $N \succeq^p M$ p-ω-saturo

$N \models \{\varphi\}'$ quindi $N \models \varphi$ quindi $M \models \varphi$.

②⇒① $M \preceq^p N \models \varphi$ basta mostrare che $M \models \{\varphi\}'$ per assurdo

prendiamo $\varphi' > \varphi$ t.c. $M \models \neg\varphi'$ esiste $\tilde{\varphi} \perp \varphi$ t.c. $\varphi \to \neg\tilde{\varphi} \to \varphi'$

$M \models \tilde{\varphi}$ quindi $N \models \tilde{\varphi}$ ↯

$\mathcal{U}$ p-saturo modello $\quad M \preceq^p \mathcal{U}$

$M \models \varphi \Rightarrow \mathcal{U} \models \varphi$

Fatto M modello

$M \models \{\varphi\}' \Longleftrightarrow \mathcal{U} \models \{\varphi\}'$

Fatto $p(x) \subseteq \mathfrak{Z}^p$ ① se $p(x) \longrightarrow \neg\varphi(x)$ allora esiste $\psi(x) \in p$ $\vee\; \psi(x) \longrightarrow \neg\varphi(x)$

$\varphi(x) \in \mathfrak{Z}^p$ ② se $p(x) \longrightarrow \varphi(x)$ per ogni $\varphi' > \varphi$ esiste $\psi(x) \in p$

$\psi(x) \longrightarrow \varphi'(x)$

Dim ① $p(x) \cup \{\varphi(x)\} \vdash \perp$ ..... compattezza

Dim (2) $p(x) \cup \{\neg\varphi(x)\} \vdash \perp$ STOP

fissiamo $\varphi' \geq \varphi$ sia $\hat{\varphi} \perp \varphi$ tale che $\varphi \to \neg\hat{\varphi} \to \varphi'$

$p(x) \cup \{\hat{\varphi}(x)\} \vdash \perp$ ---- (1) ---- $\psi(x) \to \varphi'(x)$ □

Notazione $p/c\text{-}tp(a/A) = \{\varphi(x) \in \mathfrak{F}^{p/c} : \mathcal{U} \models \varphi(a)\}$

$\mathcal{H}\text{-}tp(a/A) = \{\varphi(x) \in \mathcal{H} : \mathcal{U} \models \varphi(a)\}$

$tp(a/A) = \{\varphi(x) \in \mathcal{L} : \mathcal{U} \models \varphi(a)\}$

Fatto $p(x) \subseteq \mathfrak{F}^{p/c}(A)$ LSASE
(1) $p(x)$ è massimale consistente in $\mathcal{U}$
(2) $p(x) = p/c\text{-}tp(a/A)$ per qualche $a \in \mathcal{U}$

Dim (1) ⇒ (2) prendiamo $a \models p(x)$

$p(x) \subseteq p/c\text{-}tp(a)$

$\supseteq$ si ottiene dalla massimalità

(3) Vedremo $p(x) \mapsto tp(a/A)$ (per p)
i tipi di $\mathfrak{F}^{p}$ e $\mathcal{L}$ sono gli stessi

Dim (2) ⇒ (1) $\varphi(x) \in \mathfrak{F}^{p/c}(A) \setminus p$ allora $\neg\varphi(a)$.

sappiamo che $\{\varphi(a)\}' \leftrightarrow \varphi(a)$ allora $\neg\varphi'(a)$ per qualche $\varphi' > \varphi$

sia $\tilde{\varphi} \perp \varphi$ tale che $\varphi \to \neg\tilde{\varphi} \to \varphi'$ quindi $\hat{\varphi}(a)$

$\hat{\varphi}(x) \in p$ quindi $p(x) \to \neg\varphi(x)$.

Definizione $\mathcal{H} \subseteq \mathfrak{F}^{p/c}$ è p/c-denso se per ogni $\varphi' > \varphi \in \mathfrak{F}^{p/c}$
esiste $\psi \in \mathcal{H}$ tale che $\varphi \to \psi \to \varphi'$

Proposizione $\mathcal{H} \subseteq \mathfrak{F}^{p/c}$ p/c-denso per ogni $\varphi \in \mathfrak{F}^{p/c}$

(i) $\neg\varphi(x) \to \bigvee\{\psi(x) \in \mathcal{H} : \psi'(x) \to \neg\varphi(x),\ \psi' > \psi\}$

(ii) $\neg\varphi \leftrightarrow \bigvee\{\neg\psi \in \mathcal{H} : \neg\psi \to \neg\varphi\}$

Dim $a \models \neg\varphi(x)$ $p(x) = p/c\text{-}tp(a)$ è massimale

$p(x) \leftrightarrow p'(x) \leftrightarrow q(x) = \exists f\text{-}tp(a) \leftrightarrow q'(x)$

$q'(x) \longrightarrow \neg\varphi(x)$ $\qquad \psi'(x) \in q'$ tale che $\psi(x) \longrightarrow \neg\varphi(x)$

$\psi(a)$

(ii) $\varphi \leftrightarrow \bigwedge \{ \psi \in \mathcal{H} : \varphi \to \psi \}$

per densità

$\varphi \leftrightarrow \{\varphi\}'$