arc length s, since most concepts are defined only in terms of the derivatives of $\alpha(s)$.

It is convenient to set still another convention. Given the curve α parametrized by arc length $s \in (a, b)$, we may consider the curve β defined in $(-b, -a)$ by $\beta(-s) = \alpha(s)$, which has the same trace as the first one but is described in the opposite direction. We say, then, that these two curves differ by a *change of orientation*.

EXERCISES

1. Show that the tangent lines to the regular parametrized curve $\alpha(t) = (3t, 3t^2, 2t^3)$ make a constant angle with the line $y = 0$, $z = x$.

2. A circular disk of radius 1 in the plane xy rolls without slipping along the x axis. The figure described by a point of the circumference of the disk is called a *cycloid* (Fig. 1-7).

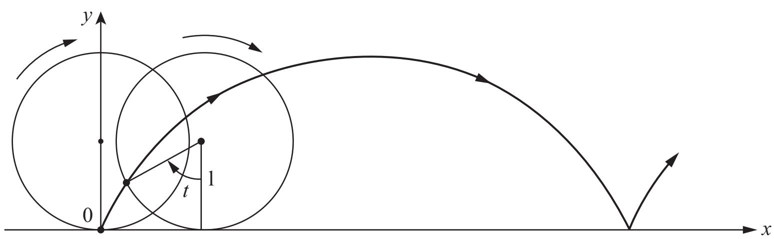

Figure 1-7. The cycloid.

***a.** Obtain a parametrized curve α: $R \to R^2$ the trace of which is the cycloid, and determine its singular points.

b. Compute the arc length of the cycloid corresponding to a complete rotation of the disk.

3. Let $0A = 2a$ be the diameter of a circle S^1 and $0y$ and AV be the tangents to S^1 at 0 and A, respectively. A half-line r is drawn from 0 which meets the circle S^1 at C and the line AV at B. On $0B$ mark off the segment $0p = CB$. If we rotate r about 0, the point p will describe a curve called the *cissoid of Diocles*. By taking $0A$ as the x axis and $0Y$ as the y axis, prove that

a. The trace of

$$\alpha(t) = \left(\frac{2at^2}{1+t^2}, \frac{2at^3}{1+t^2}\right), \qquad t \in R,$$

is the cissoid of Diocles ($t = \tan\theta$; see Fig. 1-8).

b. The origin (0, 0) is a singular point of the cissoid.

c. As $t \to \infty$, $\alpha(t)$ approaches the line $x = 2a$, and $\alpha'(t) \to 0, 2a$. Thus, as $t \to \infty$, the curve and its tangent approach the line $x = 2a$; we say that $x = 2a$ is an *asymptote* to the cissoid.

4. Let $\alpha\colon (0, \pi) \to R^2$ be given by

$$\alpha(t) = \left(\sin t, \cos t + \log \tan \frac{t}{2}\right),$$

where t is the angle that the y axis makes with the vector $\alpha'(t)$. The trace of α is called the *tractrix* (Fig. 1-9). Show that

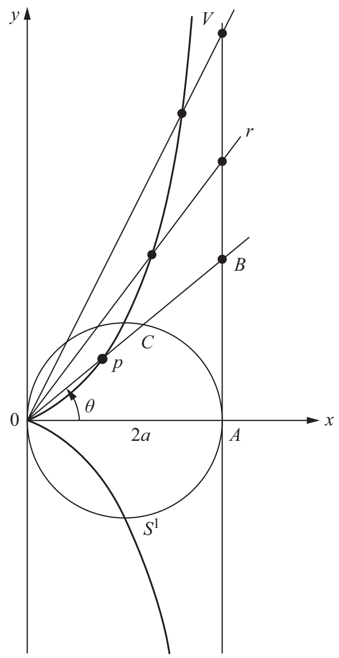

Figure 1-8. The cissoid of Diocles.

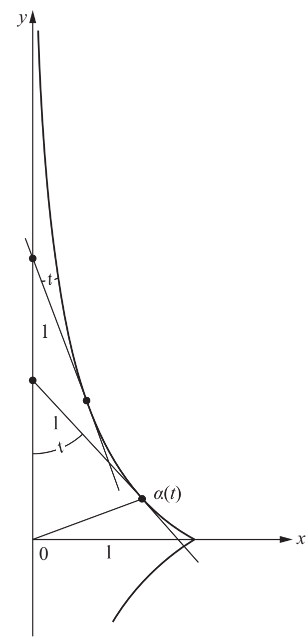

Figure 1-9. The tractrix.

a. α is a differentiable parametrized curve, regular except at $t = \pi/2$.

b. The length of the segment of the tangent of the tractrix between the point of tangency and the y axis is constantly equal to 1.

5. Let $\alpha\colon (-1, +\infty) \to R^2$ be given by

$$\alpha(t) = \left(\frac{3at}{1+t^3}, \frac{3at^2}{1+t^3}\right).$$