11. Show that the set $S = \{(x, y, z) \in R^3; z = x^2 - y^2\}$ is a regular surface and check that parts a and b are parametrizations for S:

a. $\mathbf{x}(u, v) = (u + v, u - v, 4uv), (u, v) \in R^2$.

***b.** $\mathbf{x}(u, v) = (u \cosh v, u \sinh v, u^2), (u, v) \in R^2, u \neq 0$.

Which parts of S do these parametrizations cover?

12. Show that $\mathbf{x}: U \subset R^2 \to R^3$ given by

$$\mathbf{x}(u, v) = (a \sin u \cos v, b \sin u \sin v, c \cos u), \quad a, b, c \neq 0,$$

where $0 < u < \pi$, $0 < v < 2\pi$, is a parametrization for the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

Describe geometrically the curves $u = \text{const.}$ on the ellipsoid.

***13.** Find a parametrization for the hyperboloid of two sheets $\{(x, y, z) \in R^3; -x^2 - y^2 + z^2 = 1\}$.

14. A half-line $[0, \infty)$ is perpendicular to a line E and rotates about E from a given initial position while its origin 0 moves along E. The movement is such that when $[0, \infty)$ has rotated through an angle θ, the origin is at a distance $d = \sin^2(\theta/2)$ from its initial position on E. Verify that by removing the line E from the image of the rotating line we obtain a regular surface. If the movement were such that $d = \sin(\theta/2)$, what else would need to be excluded to have a regular surface?

***15.** Let two points $p(t)$ and $q(t)$ move with the same speed, p starting from $(0, 0, 0)$ and moving along the z axis and q starting at $(a, 0, 0)$, $a \neq 0$, and moving parallel to the y axis. Show that the line through $p(t)$ and $q(t)$ describes a set in R^3 given by $y(x - a) + zx = 0$. Is this a regular surface?

16. One way to define a system of coordinates for the sphere S^2, given by $x^2 + y^2 + (z - 1)^2 = 1$, is to consider the so-called *stereographic projection* $\pi: S^2 \sim \{N\} \to R^2$ which carries a point $p = (x, y, z)$ of the sphere S^2 minus the north pole $N = (0, 0, 2)$ onto the intersection of the xy plane with the straight line which connects N to p (Fig. 2-12). Let $(u, v) = \pi(x, y, z)$, where $(x, y, z) \in S^2 \sim \{N\}$ and $(u, v) \in xy$ plane.

a. Show that $\pi^{-1}: R^2 \to S^2$ is given by

$$\pi^{-1}\begin{cases} x = \dfrac{4u}{u^2 + v^2 + 4}, \\ y = \dfrac{4v}{u^2 + v^2 + 4}, \\ z = \dfrac{2(u^2 + v^2)}{u^2 + v^2 + 4}. \end{cases}$$