18. Use the fact that if $\theta = 2\pi/m$, then

$$\sigma(\theta) = 1 + \cos^2\theta + \cdots + \cos^2(m-1)\theta = \frac{m}{2},$$

which may be proved by observing that

$$\sigma(\theta) = \frac{1}{4}\left(\sum_{v=-(m-1)}^{v=m-1} e^{2vi\theta} + 2m + 1\right)$$

and that the expression under the summation sign is the sum of a geometric progression, which yields

$$\frac{\sin(2m\theta - \theta)}{\sin\theta} = -1.$$

19. **a.** Express t and h in the basis $\{e_1, e_2\}$ given by the principal directions, and compute $\langle dN(t), h\rangle$.

b. Differentiate $\cos\theta = \langle N, n\rangle$, use that $dN(t) = -k_n t + \tau_g h$, and observe that $\langle N, b\rangle = \langle h, N\rangle = \sin\theta$, where b is the binormal vector.

20. Let S_1, S_2, and S_3 be the surfaces that pass through p. Show that the geodesic torsions of $C_1 = S_2 \cap S_3$ relative to S_2 and S_3 are equal; it will be denoted by τ_1. Similarly, τ_2 denotes the geodesic torsion of $C_2 = S_1 \cap S_3$ and τ_3 that of $S_1 \cap S_2$. Use the definition of τ_g to show that, since C_1, C_2, C_3 are pairwise orthogonal, $\tau_1 + \tau_2 = 0$, $\tau_2 + \tau_3 = 0$, $\tau_3 + \tau_1 = 0$. It follows that $\tau_1 = \tau_2 = \tau_3 = 0$.

SECTION 3-3

2. Asymptotic curves: $u = \text{const.}$, $v = \text{const.}$ Lines of curvature:

$$\log(v + \sqrt{v^2 + c^2}) \pm u = \text{const.}$$

3. $u + v = \text{const.}$ $u - v = \text{const.}$

6. **a.** By taking the line r as the z axis and a normal to r as the x axis, we have that

$$z' = \frac{\sqrt{1 - x^2}}{x}.$$

By setting $x = \sin\theta$, we obtain

$$z(\theta) = \int \frac{\cos^2\theta}{\sin\theta}\, d\theta = \log\tan\frac{\theta}{2} + \cos\theta + C.$$

If $z(\pi/2) = 0$, then $C = 0$.